**CHAPTER IV: Categorical Propositions**

**4.1 The Components of Categorical Propositions**

A proposition (or statement) is a sentence that is either true or false. A proposition that relates two classes, or categories, is called a **categorical proposition**. The classes in question are denoted respectively by the subject term and the predicate term.

**Categorical proposition** asserts that either **all or part of the class** denoted by the subject term is **included in or excluded** from the class denoted by the predicate term.

Here are some examples of categorical propositions:

Children are human being.

Snowmobiles do not belong in Yellowstone National Park.

Many of today's cell phones contain cameras.

Not all romances have a happy ending.

Oprah Winfrey publishes magazines.

The first statement asserts that the entire class of *children* is included in the class of human being, the second that the entire class of snowmobiles is excluded from the class of things that belong in Yellowstone National Park, and the third that part of the class of today's cell phones is included in the class of things that contain cameras. The last statement asserts that part of the class of romances is excluded from the class of things that have a happy ending.

Given the above facts, there are exactly four types of categorical propositions: **(1)** those that assert that the whole subject class is included in the predicate class, **(2)** those that assert that the whole subject class is excluded from the predicate class, **(3)** those that assert that part of the subject class is included in the predicate class, and **(4)** those that assert that part of the subject class is excluded from the predicate class. A categorical proposition that expresses these relations with complete clarity is called a **standard-form categorical proposition**. A categorical proposition is in standard form if and only if it is a substitution instance of one of the following four forms:

**All *S* are *P****. ---------------(*The entire subject class is included in the predicate class.)

***No S* are *P****.---------------(* The entire subject class is excluded from the predicate class.)

***Some S are P****.------------(* Part of the subject class is included in the predicate class.)

**Some S are not *P****.-------(* Part of the subject class is excluded from the predicate class.)

Many categorical propositions, of course, are not in standard form because, among other things, they do not begin with the words "all," "no," or "some." These words are called **quantifiers** because they specify how much of the subject class is included in or excluded from the predicate class. (Incidentally, in formal deductive logic the word "**some**" always means **at least one**.) The letters ***S***and ***P***stand respectively for the subject and predicate terms, and the words "**are**" and "**are not**" are called the **copula** because they link (or "couple") the subject term with the predicate term.

Consider the following example:

All members of the American Medical Association are people holding degrees from recognized academic institutions.

This standard-form categorical proposition is analyzed as follows:

*Quantifier: all*

*Subject term (S):* members of the American Medical Association

*Copula:* are

*Predicate term (P):* people holding degrees from recognized academic institutions

In resolving standard-form categorical propositions into their four components, one must keep these components separate. ***They do not overlap***. In this regard, note that "subject term" and "predicate term" do not mean the same thing in logic that "subject" and "predicate" mean in grammar. The *subject* of the example statement includes the quantifier "all," but the *subject term* does not. Similarly, the *predicate* includes the copula "are," but the *predicate term* does not.

Two additional points should be noted about standard-form categorical propositions. The first is that the form "***All S are not P" is not a standard form***. This form is ambiguous and can be rendered as either "No *S* are *P"* or "Some *S* are not *P,"* depending on the content. The second point is that there are exactly three forms of quantifiers and two forms of copulas. Other texts allow the various forms of the verb "to be" (such as "is," "is not," "will," and "will not") to serve as the copula. For the sake of uniformity, this book restricts the copula to "are" and "are not."

**4.2 Quality, Quantity, and Distribution**

Quality and quantity are *attributes of categorical propositions*. In order to see how these attributes pertain, it is useful to rephrase the meaning of categorical propositions in class terminology:

**Proposition Meaning in class notation**

All *S are P. → Every* member of the *S* class is a member of the *P* class; i.e., the *S* class is included in the *P* class.

No *S* are *P. → No* member of the *S* class is a member of the *P class;* i.e., the *S* class is excluded from the *P* class.

Some *S* are *P. →*At least one member of the *S* class is a member of the *P* class.

Some *S* are not *P. →*At least one member of the *S* class is not a member of the *P class.*

**The quality** of a categorical proposition is either **affirmative or negative** depending on whether it **affirms** (class inclusion) **or denies** (class exclusion) class membership. Accordingly:

"All *S* are *P"* and "Some *S* are *P"* **affirms** class membership andhave affirmative quality, and

"No *S* are *P"* and "Some *S* are not *P"* **denies** class membership and have negative quality.

These are called affirmative statements and negative statements, respectively.

**The quantity** of a categorical proposition is either **universal or particular**, depending on whether the statement makes a claim about ***every* member or just *some* member of the class** denoted by the **subject term**.

"All *S* are *P"* and "No *S* are *P"* each assert something about **every member** of the *S* class and thus are universal statements.

"Some *S* are *P"* and "Some *S* are not *P"* assert something about one or more members (**part**) of the *S* class and hence are particular statements.

Note that the **quantity**of a categorical proposition may be **determined**through mere inspection of the**quantifier.** *"All" and "no"* immediately imply *universal quantity, while "some"* implies *particular*. But categorical propositions have no "qualifier." In **universal** propositions the **quality** is **determined** by the **quantifier**, and in **particular** propositions it is **determined** by the **copula**.

Particular propositions mean no more and no less than the meaning assigned to them in class notation. The statement "Some *S* are *P"* does *not* imply that some *S* are not *P,* and the statement "Some *S* are not *P"* does *not* imply that some *S* are *P.* It often *happens,* of course, that substitution instances of these statement forms are both true. For example, "Some apples are red" is true, as is "Some apples are not red." But the fact that one is true does not *necessitate* that the other be true. "Some zebras are animals" is true (because at least one zebra is an animal), but "Some zebras are not animals" is false. Thus, the fact that one of these statement forms is true does not *logically imply* that the other is true, as these substitution instances clearly prove.

Since the early Middle Ages the four kinds of categorical propositions have commonly been designated by letter names corresponding to the first four vowels of the Roman alphabet: A, E, I, 0. *The universal affirmative is called an A proposition, the universal negative an E proposition, the particular affirmative an I proposition, and the particular negative an 0 proposition.* Tradition has it that these letters were derived from the first two vowels in the Latin words *affirmo* ("I affirm") and *nego* ("I deny").

**Distribution**

Unlike quality and quantity, which are attributes of *propositions,* **distribution** is an **attribute of the *terms***(subject and predicate) of propositions.

*A term is said to be distributed* if the proposition makes an assertion about every member of the class denoted by the term; otherwise, it is undistributed. Stated another way, a term is distributed if and only if the statement assigns an attribute to every member of the class denoted by the term.

Thus, if a statement asserts something about every member of the S class, then *S* is distributed; if it asserts something about every member of the *P* class, then *P* is distributed; otherwise S and *P* are undistributed.

Let us imagine that the members of the classes denoted by the subject and predicate terms of a categorical proposition are contained respectively in circles marked with the letters "S" and *"P."*

**The universal affirmative** (A) proposition*: t*he meaning of the statement form "All S are *P"* may then be represented by the following diagram:

P

S

The *S* circle is contained in the *P* circle, which represents the fact that every member of S is a member of *P.* (Of course, should S and *P* represent terms denoting identical classes, the two circles would overlap exactly.) As the diagram shows, "All *S* are *P"* makes a claim about every member of the *S* class, since the statement says that every member of S is in the *P* class. But the statement does not make a claim about every member of the *P* class, since there may be some members of *P* that are outside of *S.* Thus, by the definition of "distributed term" given above, *S* is distributed and *P* is not. In other words, for any A proposition, the subject term is distributed, and the predicate term is undistributed.

Let us now consider the **universal negative** (E) proposition. "No *S* are *P"* states that the *S* and *P* classes are separate, which may be represented as follows:

S P

This statement makes a claim about every member of *S* and every member of P. It asserts that every member of *S* is separate from every member of *P,* and also that every member of *P* is separate from every member of *S.* Accordingly, both the subject and predicate terms of E propositions are distributed.

**The particular affirmative** (I) proposition states that at least one member of S is a member of *P.* If we represent this one member of S that we are certain about by an asterisk, the resulting diagram looks like this:

\*S

P

Since the asterisk is inside the *P* class, it represents something that is simultaneously an *S* and a *P;* in other words, it represents a member of the *S* class that is also a member of the *P* class. Thus, the statement "Some *S* are *P"* makes a claim about one member (at least) of *S* and also one member (at least) of *P,* but not about all members of either class. Hence, by the definition of distribution, neither *S* nor *P* is distributed.

**The particular negative** (0) proposition asserts that at least one member of S is not a member of *P.* If we once again represent this one member of *S* by an asterisk, the resulting diagram is as follows:

\*S

P

Since the other members of *S* may or may not be outside of *P,* it is clear that the statement "Some *S* are not *P"* does not make a claim about every member of *S,* so *S* is not distributed. But, as may be seen from the diagram, the statement does assert that every member of *P* is separate and distinct from this one member of *S* that is outside the *P* circle. Thus, in O proposition, *P* is distributed and *S* is undistributed.

At this point the notion of distribution may be somewhat vague and elusive. Unfortunately, there is no simple and easy way to make the idea graphically clear. The best that can be done is to repeat some of the things that have already been said. If the proposition in question is an A type, then the subject term, whatever it may be, is distributed. If it is an E type, then both terms are distributed; if an I type, then neither; and if an 0 type, then the predicate. If a certain term is *distributed* in a proposition, this simply means that the proposition says something about every member of the class that the term denotes. If a term is *undistributed,* the proposition does not say something about every member of the class.

Finally, note that the attribute of distribution, while not particularly important to subsequent developments in this chapter, is essential to evaluate the validity of syllogisms in the next chapter.

The material of this section may now be summarized as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Proposition** | **Letter name** | **Quantity** | **Quality** | **Terms distributed** |
| All 5 are *P.* | A | universal | affirmative | *S* |
| No *S* are *P.* | E | universal | negative | *S* and *P* |
| Some *S are P.* | I | particular | affirmative | none |
| Some *S* are not *P.* | 0 | particular | negative | *P* |

**4.3 Venn Diagrams and the Modern Square of Opposition**

**Aristotle and Boole**

Before we can address the two main topics of this section, we must look further at the meaning of universal (A and E) propositions. Consider these two A propositions:

All Tom Cruise's movies are hits.

All unicorns are one-horned animals.

The first proposition implies that Tom Cruise has indeed made some movies. In other words, the statement has existential import. It implies that one or more things denoted by the subject term actually exist. On the other hand, no such implication is made by the statement about unicorns. The statement is true, because unicorns, by definition, have a single horn. But the statement does not imply that unicorns actually exist.

Thus, the question arises: Should universal propositions be interpreted as implying that the things talked about actually exist? Or should they be interpreted as implying no such thing? In response to this question, logicians have taken two different approaches. Aristotle held that universal propositions about existing things have existential import. In other words, such statements imply the existence of the things talked about:

**Aristotelian standpoint**

All ostriches are birds. → Implies the existence of ostriches.

No pine trees are maples. → Implies the existence of pine trees.

All satyrs are vile creatures.→ Does not imply the existence of satyrs.

The first two statements have existential import because their subject terms denote actually existing things. The third statement has no existential import, because satyrs do not exist.

On the other hand, the nineteenth-century logician George Boole held that no universal propositions have existential import. Such statements never imply the existence of the things talked about:

**Boolean standpoint**

All trucks are vehicles. →Does not imply the existence of trucks.

No roses are daisies. →Does not imply the existence of roses.

All werewolves are monsters. →Does not imply the existence of werewolves.

We might summarize these results by saying that the Aristotelian standpoint is "open" to existence. When things exist, the Aristotelian standpoint recognizes their existence, and universal statements about those things have existential import. In other words, existence counts for something. On the other hand, the Boolean standpoint is "closed" to existence. When things exist, the Boolean standpoint does not recognize their existence, and universal statements about those things have no existential import.

The Aristotelian standpoint differs from the Boolean standpoint only with regard to universal (A and E) propositions. The two standpoints are identical with regard to particular (I and 0) propositions. Both the Aristotelian and the Boolean standpoints recognize that particular propositions make a positive assertion about existence. For example, from both standpoints the statement "Some cats are animals" asserts that at least one cat exists, and that cat is an animal. Also, from both standpoints, "Some fish are not mammals" asserts that at least one fish exists, and that fish is not a mammal. Thus, from both standpoints, the word "some" implies existence.\*

**Note**: ***Adopting either the Aristotelian or the Boolean standpoint amounts to, accepting a set of ground rules for interpreting the meaning of universal propositions***.

Because the Boolean standpoint is neutral about existence, it is simpler than the Aristotelian standpoint, which recognizes existential implications. For this reason, we will direct our attention first to arguments considered from the Boolean standpoint. Later, we will extend our treatment to the Aristotelian standpoint.

\*In ordinary language, the word "some" occasionally implies something less than actual existence. For example, the statement "Some unicorns are tenderhearted" does not seem to suggest that unicorns actually exist, but merely that among the group of imaginary things called "unicorns," there is a subclass of tenderhearted ones. In the vast majority of cases, however, "some" in ordinary language implies existence. The logical "some" conforms to these latter uses.

**Venn Diagrams**

From the Boolean standpoint, the four kinds of categorical propositions have the following meaning. Notice that the first two (universal) propositions imply nothing about the existence of the things denoted by *S:*

All S are P. = No members of S are outside P.

No S are P. = No members of S are inside P.

Some S are P. = At least one S exists, and that S is a P.

Some S are not P. = At least one S exists, and that S is not a P.

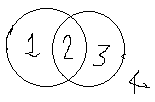
Adopting this interpretation of categorical propositions, the nineteenth-century logician John Venn developed a system of diagrams to represent the information they express. These diagrams have come to be known as Venn diagrams.

**A Venn diagram** is an arrangement of overlapping circles in which each circle represents the class denoted by a term in a categorical proposition. Because every categorical proposition has exactly two terms, the Venn diagram for a single categorical proposition consists of two overlapping circles. Each circle is labeled so that it represents one of the terms in the proposition. Unless otherwise required, we adopt the convention that the left-hand circle represents the subject term, and the right-hand circle the predicate term. Such a diagram looks like this:

S P

The members of the class denoted by each term should be thought of as situated inside the corresponding circle. Thus, the members of the *S* class (if any such members exist) are situated inside the *S* circle, and the members of the *P* class (if any such members exist) are situated inside the *P* circle. If any members are situated inside the area where the two circles overlap, then such members belong to both the *S* class and the *P* class. Finally, if any members are situated outside both circles, they are members of neither *S* nor *P.*

Suppose, for example, that the *S* class is the class of Americans and the *P* class is the class of farmers. Then, if we use numerals to identify the four possible areas, the diagram looks like this:



Anything in the area marked "1" is an American but not a farmer, anything in the area marked "2" is both an American and a farmer, and anything in the area marked "3" is a farmer but not an American. The area marked "4" is the area outside both circles is neither a farmer nor an American.

We can now use Venn diagrams to represent the information expressed by the four kinds of categorical proposition. To do this we make a certain kind of mark in a diagram. **Two kinds of marks are used**: shading an area and placing an X in an area. **Shading** an area means that the shaded area is empty,\* and **placing an X** in an area means that at least one thing exists in that area. The X may be thought of as representing that one thing. **If no mark appears** in an area, this means that nothing is known about that area; it may contain members or it may be empty. Shading is always used to represent the content of universal (A and E) propositions, and placing an X in an area is always used to represent the content of particular (I and 0) propositions. The content of the four kinds of categorical propositions is represented as follows:

A: All S are P

S p

E: No S are P

S P

I: Some S are P

S P

O: Some S are not P

S P

Recall that the A proposition asserts that no members of *S* are outside *P.* This is represented by shading the part of the *S* circle that lies outside the *P* circle. The E proposition asserts that no members of *S* are inside *P.* This is represented by shading the part of the *S* circle that lies inside the *P* circle. The I proposition asserts that at least one *S* exists and that *S* is also a *P.* This is represented by placing an X in the area where the *S* and *P* circles overlap. This X represents an existing thing that is both an *S* and a *P.* Finally, the 0 proposition asserts that at least one *S* exists, and that *S* is not a *P.* This is represented by placing an X in the part of the *S* circle that lies outside the *P* circle. This X represents an existing thing that is an *S* but not a *P.*

Because there is no X in the diagrams that represent the universal propositions, these diagrams say nothing about existence. For example, the diagram for the A proposition merely asserts that nothing exists in the part of the *S* circle that lies outside the *P* circle. The areas where the two circles overlap and the part of the *P* circle that lies out-side the *S* circle contain no marks at all. This means that something might exist in these areas, or they might be completely empty. Similarly, in the diagram for the E proposition, no marks appear in the left-hand part of the *S* circle and the right-hand part of the *P* circle. This means that these two areas might contain something or, on the other hand, they might not.

**The Modern Square of Opposition**

Let us compare the diagram for the A proposition with the diagram for the 0 proposition. The diagram for the A proposition asserts that the left-hand part of the *S* circle is empty, whereas the diagram for the 0 proposition asserts that this same area is not empty. These two diagrams make assertions that are the exact opposite of each other. As a result, their corresponding statements are said to contradict each other. Analogously, the diagram for the E proposition asserts that the area where the two circles overlap is empty, whereas the diagram for the I proposition asserts that the area where the two circles overlap is not empty. Accordingly, their corresponding propositions are also said to contradict each other. This relationship of mutually contradictory pairs of propositions is represented in a diagram called the modern square of opposition. This diagram, which arises from the modern (or Boolean) interpretation of categorical propositions, is represented as follows:

**A** Logically undetermined **E**

|  |
| --- |
|  |

Logicallyundetermined Logically undetermined   **I** Logically undetermined **O**

If two propositions are related by the contradictory relation, they necessarily have opposite truth value. Thus, if certain A proposition is given as true, the corresponding 0 proposition must be false. Similarly, if certain I proposition is given as false, the corresponding E proposition must be true. But no other inferences are possible. In particular, given the truth value of an A or 0 proposition, nothing can be determined about the truth value of the corresponding E or I propositions. These propositions are said to have logically undetermined truth value. Like all propositions, they do have a truth value, but logic alone cannot determine what it is. Similarly, given the truth value of an E or I proposition, nothing can be determined about the truth value of the corresponding A or 0 propositions. They, too, are said to have logically undetermined truth value.

**Testing Immediate Inferences**

Since the modern square of opposition provides logically necessary results, we can use it to test certain arguments for validity. We begin by assuming the premise is true, and we enter the pertinent truth value in the square. We then use the square to compute the truth value of the conclusion. If the square indicates that the conclusion is true, the argument is valid; if not, the argument is invalid. Here is an example:

Some trade spies are not masters at bribery.

Therefore, it is false that all trade spies are masters at bribery.

Arguments of this sort are called immediate inferences because they have only one premise. Instead of reasoning from one premise to the next, and then to the conclusion, we proceed immediately to the conclusion. To test this argument for validity, we begin by assuming that the premise, which is an O proposition, is true, and we enter this truth value in the square of opposition. We then use the square to compute the truth value of the corresponding A proposition. By the contradictory relation, the A proposition is false. Since the conclusion claims that the A proposition is false, the conclusion is true, and therefore the argument is valid. Arguments that are valid from the Boolean standpoint are said to be unconditionally valid because they are valid regardless of whether their terms refer to existing things.

Note that the conclusion of this argument has the form "It is false that all *S* are *P."* Technically, statements of this type are not standard-form propositions because, among other things, they do not begin with a quantifier. To remedy this difficulty we adopt the convention that statements having this form are equivalent to " 'All *S* are *P'* is false." Analogous remarks apply to the negations of the E, I, and 0 statements.

Here is another example:

It is false that all meteor showers are common spectacles. Therefore, no meteor showers are common spectacles.

We begin by assuming that the premise is true. Since the premise claims that an A proposition is false, we enter "false" into the square of opposition. We then use the square to compute the truth value of the corresponding E proposition. Since there is no relation that links the A and E propositions, the E proposition has undetermined truth value. Thus, the conclusion of the argument has undetermined truth value, and the argument is invalid.

We can also use Venn diagrams to test immediate inferences for validity. To do so we begin by using letters to represent the terms, and we then draw Venn diagrams for the premise and conclusion. If the information expressed by the conclusion diagram is contained in the premise diagram, the argument is valid; if not, it is invalid. Here is the symbolized form of the trade spies argument that we tested earlier.

Some *T* are not *M.*

Therefore, it is false that all T are *M.*

The next step is to draw two Venn diagrams, one for the premise and the other for the conclusion. The premise diagram is easy; all we need do is place an X in the left-hand part of the *T* circle. But drawing the conclusion is a bit more complicated. First we consider how we would diagram "All *T* are M." We would shade the left-hand part of the *T* circle. But since the conclusion asserts that "All *T* are M" is *false,* we do just the opposite: we place an X in the left-hand part of the *T* circle. Here are the completed diagrams:

x

x

Some T are M It is false that all T are M

T M T M

To evaluate the argument, we look to see whether the information expressed by the conclusion diagram is also expressed by the premise diagram. The conclusion diagram asserts that something exists in the left-hand part of the *T* circle. Since this information is also expressed by the premise diagram, the argument is valid. In this case, the diagram for the conclusion is identical to the diagram for the premise, so it is clear that premise and conclusion assert exactly the same thing. However, as we will see in the next Sections, for an argument to be valid, it is not necessary that premise and conclusion assert exactly the same thing. It is only necessary that the premise assert *at least as much* as the conclusion.

Here is the symbolized version of the second argument evaluated earlier:

It is false that all *M* are C. Therefore, no M are *C.*

In diagramming the premise, we do just the opposite of what we would do to diagram "All M are C." Instead of shading the left-hand part of the M circle, we place an X in that area. For the conclusion we shade the area where the two circles overlap:

x

It is false that all *M* are C No M are C

M C M C

Here, the conclusion diagram asserts that the overlap area is empty. Since this information is not contained in the premise diagram, the argument is invalid.

We conclude with a special example:

All cell phones are wireless devices. Therefore, some cell phones are wireless devices.

The completed Venn diagrams are as follows:

All C are W Some C are W

C W C W

The information of the conclusion diagram is not contained in the premise diagram, so the argument is invalid. However, if the premise were interpreted as having existential import, then the C circle in the premise diagram would not be empty. Specifically, there would be members in the overlap area. This would make the argument valid.

Arguments of this sort are said to commit the **existential fallacy**. From the Boolean standpoint, the **existential fallacy** is a formal fallacy that is committed whenever an argument is invalid merely because the premise is interpreted as lacking existential import. In Section 4.5, additional forms of this fallacy will be explored. For now, we can identify two forms of the fallacy that arise from the Boolean standpoint:

**Existential fallacy**

All A are B. No A are B.

Therefore, some A are B. Therefore, some A are not B.

**Exercise 4.3: Omitted**

**4.4 Conversion, Obversion, and Contraposition**

For a preliminary glimpse into the content of this section, consider the statement "No dogs are cats." This statement claims that the class of dogs is separated from the class of cats. But the statement "No cats are dogs" claims the same thing. Thus, the two statements have the same meaning and the same truth value. For another example, consider the statement "Some dogs are not retrievers." This statement claims there is at least one dog outside the class of retrievers. But the statement "Some dogs are non-retrievers" claims the same thing, so again, the two statements have the same meaning and the same truth value.

Conversion, obversion, and contraposition are operations that can be performed on a categorical proposition, resulting in a new statement that may or may not have the same meaning and truth value as the original statement. Venn diagrams are used to determine how the two statements relate to each other.

**Conversion**

Conversion consists in switching the subject term with the predicate term. For example, if the statement "No foxes are buffalos" is converted, the resulting statement is "No buffalo are foxes." This new statement is called the *converse* of the given statement. To see how the four types of categorical propositions relate to their converse, compare the following sets of Venn diagrams:

Given statements Converse

All A are B All B are A

A B A B

No A are B No B are A

A B A B

Some A are B Some B are A

A B A B

Some A are not B Some B are not A

A B A B

If we examine the diagram for the E statement, we see that it is identical to that of its converse. Also, the diagram for the I statement is identical to that of its converse. This means that the E statement and its converse are logically equivalent, and the I statement and its converse are logically equivalent. Two statements are said to be logically equivalent statements when they necessarily have the same truth value. Thus, converting an E or I statement gives a new statement that always has the same truth value (and the same meaning) as the given statement. These equivalences are strictly proved by the Venn diagrams for the E and I statements.

Converse (quantifier) S copula P

On the other hand, the diagram for the A statement is clearly not identical to the diagram for its converse, and the diagram for the 0 statement is not identical to the diagram for its converse. Also, these pairs of diagrams are not the exact opposite of each other, as is the case with contradictory statements. This means that an A statement and its converse are logically unrelated as to truth value, and an 0 statement and its converse are logically unrelated as to truth value. In other words, converting an A or 0 statements gives a new statement whose truth value is logically undetermined in relation to the given statement. The converse of an A or 0 statements does have a truth value, of course, but logic alone cannot tell us what it is.

Because conversion yields necessarily determined results for E and I statements, it can be used as the basis for immediate inferences having these types of statements as premises. The following argument forms are **valid**:

No A are B. Therefore, no B are A.

Some A are B. Therefore, some B are A.

Since the conclusion of each argument form necessarily has the same truth value as the premise, if the premise is assumed true, it follows necessarily that the conclusion is true. On the other hand, the next two argument forms are **invalid**. Each commits the fallacy of illicit conversion:

All A are B. Therefore, all B are A.

Some A are not B. Therefore, some B are not A.

Here are two examples of arguments that commit the fallacy of illicit conversion:

All cats are animals. (True) Therefore, all animals are cats. (False)

Some animals are not dogs. (True) Therefore, some dogs are not animals. (False)

**Obversion**

Obversion requires two steps: (1) changing the quality (without changing the quantity),

and (2) replacing the predicate with its term complement.

The first step consists in changing "No *S* are *P"* to "All *S* are *P"* and vice versa, and changing "Some *S* are *P"* to "Some *S* are not *P"* and vice versa.

The second step requires understanding the concept of *class complement.* **The complement of a class** is the group consisting of everything outside the class. For example, the complement of the class of dogs is the group that includes everything that is not a dog (cats, fish, trees, and so on). The term complement is the word or group of words that denotes the class complement. For terms consisting of a single word, the term complement is usually formed by simply attaching the prefix "non" to the term. Thus, the complement of the term "dog" is "non-dog," the complement of the term "ox" is "non-ox," and so on.

The relationship between a term and its complement can be illustrated by a Venn diagram. For example, if a single circle is allowed to represent the class of dogs, then everything outside the circle represents the class of non-dogs:

Dogs

non-dogs

To see obverse, if we are given the statement "All horses are animals," then the obverse is "No horses are non-animals"; and if we are given the statement "Some trees are apples," then the obverse is "Some trees are not non-apples." To see how the four types of categorical propositions relate to their obverse, compare the following sets of Venn diagrams:

**Given statements Obverse**

All A are B No A are non-B

A B A B

No A are B All A are non-B

A B A B

Some A are B Some A are not non-B

A B A B

Some A are not B Some A are non-B

A B A B

To see how the obverse diagrams are drawn, keep in mind that "non-B" designates the area outside the *B* circle. Thus, "No A are non-B" asserts that the area where *A* overlaps non-B is empty. This is represented by shading the left-hand part of the A circle. "All A are non-B" asserts that all members of A are outside *B.* This means that no members of A are inside *B,* so the area where A overlaps *B* is shaded. "Some A are not non-B" asserts that at least one member of A is not outside *B.* This means that at least one member of A is inside *B,* so an X is placed in the area where A and *B* overlap. Finally, "Some A are non-B" asserts that at least one member of A is outside *B,* so an X is placed in the left-hand part of the A circle.

If we examine these pairs of diagrams, we see that the diagram for each given statement form is identical to the diagram for its obverse. This means that each of the four types of categorical proposition is logically equivalent to (and has the same meaning as) its obverse. Thus, if we obvert an A statement that happens to be true, the resulting statement will be true; if we obvert an 0 statement that happens to be false, the resulting statement will be false, and so on.

It is easy to see that if a statement is obverted and then obverted again, the resulting statement will be identical to the original statement. For example, the obverse of "All horses are animals" is "No horses are non-animals." To obvert the latter statement we again change the quality ("no" switches to "all") and replace "non-animals" with its term complement. The term complement is produced by simply deleting the prefix "non." Thus, the obverse of the obverse is "All horses are animals."

When a term consists of more than a single word, more ingenuity is required to form its term complement. For example, if we are given the term "animals that are not native to America," it would not be appropriate to form the term complement by writing "non-animals that are not native to America." Clearly it would be better to write "animals native to America." Even though this is technically not the complement of the given term, the procedure is justified if we allow a reduction in the scope of discourse. This can be seen as follows. Technically the term complement of "animals that are not native to America" denotes all kinds of things such as ripe tomatoes, battle-ships, gold rings, and so on. But if we suppose that we are talking *only* about animals (that is, we reduce the scope of discourse to animals), then the complement of this term is "animals native to America."

As is the case with conversion, obversion can be used to supply the link between the premise and the conclusion of immediate inferences. The following argument forms are valid:

1. All A are B. 3. Some A are B.

Therefore, no A are non-B. Therefore, some A are not non-B.

1. No A are B. 4. Some A are not B.

Therefore, all A are non-B. Therefore, some A are non-B.

Because the conclusion of each argument form necessarily has the same truth value as its premise, if the premise is assumed true, it follows necessarily that the conclusion is true.

**Contraposition**

Contraposition requires two steps: (1) switching the subject and predicate terms and

(2) replacing the subject and predicate terms with their term complements. For example, if the statement "All goats are animals" is contraposed, the resulting statement is "All non-animals are non-goats." This new statement is called the contra-positive of the given statement. To see how all four types of categorical propositions relate to their contrapositive, compare the following sets of diagrams:

All A are B All non-B are non-A

A B A B

No A are B No non-B are non-A

A B A B

Some A are B Some non-A are non-B

A B A B

Some A are not B Some non-B are not non-A

A B A B

To see how the first diagram on the right is drawn, remember that "non-A" designates the area outside A. Thus, "All non-B are non-A" asserts that all members of non-B are outside A. This means that no members of non-B are inside A. Thus, we shade the area where non-B overlaps A. "No non-B are non-A" asserts that the area where non-B overlaps non-A is empty. Since non-B is the area outside the *B* circle and non-A is the area outside the A circle, the place where these two areas overlap is the area outside both circles. Thus, we shade this area. "Some non-B are non-A" asserts that something exists in the area where non-B overlaps non-A. Again, this is the area outside both circles, so we place an X in this area. Finally, "Some non-B are not non-A" asserts that at least one member of non-B is outside non-A. That is, at least one member of non-B is inside A, so we place an X in the area where non-B overlaps A.

Now, inspection of the diagrams for the A and 0 statements reveals that they are identical to the diagrams of their contrapositive. Thus, the A statement and its contrapositive are logically equivalent (and have the same meaning), and the 0 statement and its contrapositive are logically equivalent (and have the same meaning). On the other hand, the diagrams of the E and I statements are neither identical to nor the exact opposite of the diagrams of their contrapositives. This means that contraposing an E or I statement gives a new statement whose truth value is logically undetermined in relation to the given statement.

As with conversion and obversion, contraposition may provide the link between the premise and the conclusion of an argument. The following argument forms are **valid**:

All A are B. Some A are not B.

Therefore, all non-B are non-A. Therefore, some non-B are not non-A.

On the other hand, the following argument forms are **invalid**. Each commits the fallacy **of illicit**

**contraposition:**

Some A are B. No A are B.

Therefore, some non-B are non-A. Therefore, no non-B are non-A.

Here are two examples of arguments that commit the fallacy of **illicit contraposition**:

No dogs are cats. (True) Some animals are non-cats. (True)

Therefore, no non-cats are non-dogs. (False) Therefore, some cats are non-animals. (False)

In regard to the first argument, an example of something that is both a non-cat and a non-dog is a pig. Thus, the conclusion implies that no pigs are pigs, which is false. In regard to the second argument, if both premise and conclusion are obverted, the premise becomes "Some animals are not cats," which is true, and the conclusion be-comes "Some cats are not animals," which is false.

Both illicit conversion and illicit contraposition are formal fallacies: They can be detected through mere examination of the form of an argument.

Finally, note that the Boolean interpretation of categorical propositions has prevailed throughout this section. This means that the results obtained are unconditional, and they hold true regardless of whether the terms in the propositions denote actually existing things. Thus, they hold for propositions about unicorns and leprechauns just as they do for propositions about dogs and animals. These results are summarized in the following table.

**Conversion:** Switch subject and predicate terms.

**Given statement Converse Truth value**

**E:** No *S* are *P*. No *P* are *S*. Same truth value as given statement

**I:** Some *S* are *P*. Some *P* are *S*. Same truth value as given statement

**A:** All *S* are *P*. All *P* are *S*. Undetermined truth value statement

**O:** Some *S* are not *P*. Some *P* are not *S*. Undetermined truth value statement

**Obversion:** Change quality, replace predicate with term complement.

**Given statement Obverse Truth value**

**A:** All *S* are *P*. No *S* are non-*P*. Same truth value as given statement

**E:** No *S* are *P*. All *S* are non-*P*. Same truth value as given statement

**I:** Some *S* are *P*. Some *S* are not non-*P*. Same truth value as given statement

**O:** Some *S* are not *P*. Some *S* are non-*P.* Same truth value as given statement

**Contraposition:** Switch subject and predicate terms, and replace each with its term complement.

**Given statement Contrapositive Truth value**

**A:** All *S* are *P*. All non-*P* are non-*S*. Same truth value as given statement

**O:** Some *S* are not *P*. Some non-*P* are not non-*S*. Same truth value as given statement

**E:** No *S* are *P*. No non-*P* are non-*S*. Undetermined truth value

**I:** Some *S* are *P*. Some non-*P* are non-*S*. Undetermined truth value

**4.5 The Traditional Square of Opposition**

In Section 4.3 we adopted the Boolean standpoint, and we saw how the modern square of opposition applies regardless of whether the propositions refer to actually existing things. In this section, we adopt the Aristotelian standpoint, which recognizes that universal propositions about existing things have existential import. For such propositions the traditional square of opposition becomes applicable. Like the modern square, the traditional square of opposition is an arrangement of lines that illustrates logically necessary relations among the four kinds of categorical propositions. However, because the Aristotelian standpoint recognizes the additional factor of existential import, the traditional square supports more inferences than does the modern square. It is represented as follows:

A Contrary E

|  |
| --- |
|  |

I Sub-contrary O

The four relations in the traditional square of opposition may be characterized as follows:

**Contradictory** \_ opposite truth value

**Contrary** \_ at least one is false (not both true)

**Sub-contrary** \_ at least one is true (not both false)

**Sub-alternation** \_ truth flows downward, and falsity flows upward

The **contradictory** relation is the same as that found in the modern square. Thus, if a certain **A** proposition is given as true, the corresponding **O** proposition is false, and vice versa, and if a certain **A** proposition is given as false, the corresponding **O** proposition is true, and vice versa. The same relation holds between the **E** and **I** propositions. The contradictory relation thus expresses complete opposition between propositions.

The **contrary** relation is the relation between **A** and **E** propositions. It differs from the contradictory in that it expresses only partial opposition. Thus, if a certain **A** proposition is given as true, the corresponding **E** proposition is false (because at least one must be false), and if an **E** proposition is given as true, the corresponding **A** proposition is false. But if an **A** proposition is given as false, the corresponding **E** proposition could be *either* true or false without violating the ‘‘at least one is false’’ rule. In this case, the **E** proposition has logically undetermined truth value. Similarly, if an **E** proposition is given as false, the corresponding **A** proposition has logically undetermined truth value.

These results are borne out in ordinary language. Thus, if we are given the actually true **A** proposition ‘‘All cats are animals,’’ the corresponding **E** proposition ‘‘No cats are animals’’ is false, and if we are given the actually true **E** proposition ‘‘No cats are dogs,’’ the corresponding **A** proposition ‘‘All cats are dogs’’ is false. Thus, the **A** and **E** propositions cannot both be true. However, they can both be false. ‘‘All animals are cats’’ and ‘‘No animals are cats’’ are both false.

The **subcontrary** relation is the relation between **I** and **O** propositions. It also expresses a kind of partial opposition. If a certain **I**proposition is given as false, the corresponding **O** proposition is true (because at leastone must be true), and if an **O** proposition is given as false, the corresponding **I** proposition is true. But if either an **I** or an **O** proposition is given as true, then thecorresponding proposition could be either true or false without violating the ‘‘at least one is true’’ rule. Thus, in this case the corresponding proposition would have logically undetermined truth value.

Again, these results are borne out in ordinary language. If we are given the actually false **I** proposition ‘‘Some cats are dogs,’’ the corresponding **O** proposition ‘‘Some cats are not dogs’’ is true, and if we are given the actually false **O** proposition ‘‘Some cats are not animals,’’ the corresponding **I** proposition ‘‘Some cats are animals’’ is true. Thus, the **I** and **O** propositions cannot both be false, but they can both be true. ‘‘Some animals are cats’’ and ‘‘Some animals are not cats’’ are both true.

The **subalternation** relation is the relation between **A** and **I** propositions on the one hand and **E** and **O** propositions, on the other. It is represented by two arrows: a downward arrow marked with the letter ‘‘T’’ (true), and an upward arrow marked with an ‘‘F’’ (false). These arrows can be thought of as pipelines through which truth values ‘‘flow.’’ The downward arrow ‘‘transmits’’ only truth, and the upward arrow only falsity. Thus, if an **A** proposition is given as true, the corresponding **I** proposition is true also, and if an **I** proposition is given as false, the corresponding **A** proposition is false. But if an **A** proposition is given as false, this truth value cannot be transmitted downward, so thecorresponding **I** proposition will have logically undetermined truth value. Conversely,if an **I** proposition is given as true, this truth value cannot be transmitted upward, sothe corresponding **A** proposition will have logically undetermined truth value. Analogousreasoning prevails for the subalternation relation between the **E** and **I** propositions. To remember the direction of the arrows for subalternation, imagine that truth descends from "above," and falsity rises up from "below?'

Now that we have explained these four relations individually, let us see how they can be used together to determine the truth values of corresponding propositions. When using the square to compute more than one truth value the first rule of thumb that we should keep in mind is always to use contradiction first. Now, let us suppose that we are told that the nonsensical proposition ‘‘All adlers are bobkins’’ is true. Suppose further that adlers actually exist, so according to traditional square of opposition:

‘‘Some adlers are not bobkins’’ is false (by the contradictory relation).

‘‘No adlers are bobkins’’ is false (by either the contrary or the subalternation relation).

‘‘Some adlers are bobkins’’ is true (by either contradictory, subalternation, or subcontrary).

Next, let us see what happens if we assume that ‘‘All adlers are bobkins’’ is false.

‘‘Some adlers are not bobkins’’ is true (the contradictory relation),

“No adlers are bobkins” is logically undetermined

‘‘Some adlers are bobkins’’ is logically undetermined.

This result illustrates two more rules of thumb. Assuming that we always use the contradictory relation first, if one of the remaining relations yields a logically undetermined truth value, the others will as well. The other rule is that whenever one statement turns out to have logically undetermined truth value, its contradictory will also. Thus, statements having logically undetermined truth value will always occur in pairs, at opposite ends of diagonals on the square.

**Testing Immediate Inferences**

Next, let us see how we can use the traditional square of opposition to test immediate inferences for validity. Here is an example:

All Swiss watches are true works of art.

Therefore, it is false that no Swiss watches are true works of art.

To evaluate this argument, we begin, as usual, by assuming the premise is true. Since the premise is an **A** proposition, by the contrary relation the corresponding **E** proposition is false. But this is exactly what the conclusion says, so the argument is valid.

Here is another example:

Some viruses are structures that attack T-cells.

Therefore, some viruses are not structures that attack T-cells.

Here the premise and conclusion are linked by the subcontrary relation. According to that relation, if the premise is assumed true, the conclusion has logically undetermined truth value, and so the argument is invalid. It commits the formal fallacy of **illicit** **subcontrary.** Analogously, arguments that depend on an incorrect application of the contrary relation commit the formal fallacy of **illicit contrary,** and arguments that depend on an illicit application of subalternation commit the formal fallacy of **illicit** **subalternation.** Some forms of these fallacies are as follows:

**Illicit contrary**

1. It is false that all *A* are *B.* 2. It is false that no *A* are *B.*

Therefore, no *A* are *B.* Therefore, all *A* are *B.*

**Illicit subcontrary**

1.Some *A* are *B. 2.* Some *A* are not *B*

Therefore, it is false that some *A* are not *B.* Therefore, some *A* are *B.*

**Illicit subalternation**

1.Some *A* are not *B.* 2. It is false that all *A* are *B*

Therefore, no *A* are *B.* Therefore, it is false that some *A* are *B.*

Cases of the incorrect application of the contradictory relation are so infrequent that an ‘‘illicit contradictory’’ fallacy is not usually recognized.

As we saw at the beginning of this section, for the traditional square of opposition to apply, the Aristotelian standpoint must be adopted, and the propositions to which it is applied must assert something about actually existing things. The question may now be asked, what happens when the Aristotelian standpoint is adopted but the propositions are about things that do not exist? The answer is that another fallacy, **the existential fallacy**, is committed. From the Aristotelian standpoint the existential fallacy is committed whenever contrary, subcontrary, and subalternation are used (in an otherwise correct way) with propositions about things that do not exist. The existential fallacy occurs only in connection with these three relations. It does not occur in connection with the contradictory relation, which holds in the same way with nonexisting things as it does with existing things. Also, for the same reason, it does not occur in connection with conversion, obversion, and contraposition. The following inferences commit the existential fallacy:

1. All witches who fly on broomsticks are fearless women.

Therefore, some witches who fly on broomsticks are fearless women.

1. No wizards with magical powers are malevolent beings.

Therefore, it is false that all wizards with magical powers are malevolent beings.

The first depends on an otherwise correct use of the subalternation relation, and the second on an otherwise correct use of the contrary relation. If flying witches and magical wizards actually existed, both arguments would be valid. But since they do not exist, both arguments are invalid and commit the existential fallacy.

In summary, **the existential fallacy can be committed in either of two ways**. As we saw in Section 4.3, from the Boolean standpoint, the existential fallacy is committed whenever an argument is invalid merely because the premise is interpreted as lacking existential import. Thus, the argument "All cats are animals; therefore, some cats are animals" commits the existential fallacy from the Boolean standpoint. On the other hand, from the Aristotelian standpoint, the existential fallacy is committed whenever the validity of an argument depends on existential import and the requisite things do not exist. Thus, the argument "All unicorns are animals; therefore, some unicorns are animals" commits the existential fallacy from the Aristotelian standpoint. Any argument that commits the existential fallacy from the Aristotelian standpoint also commits it from the Boolean standpoint, but the converse obviously is not true.

**Existential fallacy examples- Two standpoints**

All cats are animals →Boolean: Invalid, existential fallacy

Some cats are animals Aristotelian: Valid

All unicorns are animals →Boolean: Invalid, existential fallacy

Some unicorns are animals Aristotelian: Invalid, existential fallacy

The phrase conditionally valid applies to an argument after the Aristotelian stand-point has been adopted and we are not certain if the subject term of the premise denotes actually existing things. For example, the following argument is conditionally valid:

All students who failed the exam are students on probation.

Therefore, some students who failed the exam are students on probation.

The validity of this argument rests on whether there were in fact any students who failed the exam. The argument is either valid or invalid, but we lack sufficient information about the meaning of the premise to tell which the case is. Once it becomes known that there are indeed some students who failed the exam, we can assert that the argument is valid from the Aristotelian standpoint. But if there are no students who failed the exam, the argument is invalid because it commits the existential fallacy.

Similarly, all argument *forms* that depend on valid applications of contrary, subcontrary, and subalternation are conditionally valid because we do not know if the letters in the propositions denote actually existing things. For example, the following argument form, which depends on the contrary relation, is conditionally valid:

All *A* are *B.*

Therefore, it is false that no *A* are *B.*

If "dogs" and "animals" are substituted in place of A and *B,* respectively, the resulting argument is valid. But if "unicorns" and "animals" are substituted, the resulting argument is invalid because it commits the existential fallacy. In Section 4.3, we noted that all arguments (and argument forms) that are valid from the Boolean standpoint are *unconditionally valid.* They are valid regardless of whether their terms denote actually existing things.

Now that we have seen how the traditional square of opposition, by itself, is used to test arguments for validity, let us see how it can be used together with the operations of conversion, obversion, and contraposition to prove the validity of arguments that are given as valid. Suppose we are given the following valid argument:

All inappropriate remarks are faux pas.

Therefore, some faux pas are not appropriate remarks.

To prove this argument valid, we select letters to represent the terms, and then we use some combination of conversion, obversion, and contraposition together with the traditional square to find the intermediate links between premise and conclusion:

All non-A are *F.* (assumed true)

Some non-A are *F.* (true by subalternation)

Some *F* are non-A. (true by conversion)

Therefore, some *F* are not *A.* (true by obversion)

The premise is the first line in this proof, and each succeeding step is validly derived from the one preceding it by the relation written in parentheses at the right. Since the conclusion (which is the last step) follows by a series of three necessary inferences, the argument is valid.

Various strategies can be used to construct proofs such as this, but one useful procedure is first to concentrate on obtaining the individual terms as they appear in the conclusion, then to attend to the order of the terms, and finally to use the square of opposition to adjust quality and quantity. As the example proof illustrates, however, variations on this procedure are sometimes necessary. The fact that the predicate of the conclusion is "A," while "non-A" appears in the premise, leads us to think of obversion. But using obversion to change "non-A" into "A" requires that the "non-A" in the premise be moved into the predicate position via conversion. The latter operation, however, is valid only on E and I statements, and the premise is an A statement. The fact that the conclusion is a particular statement suggests subalternation as an inter-mediate step, thus yielding an I statement that can be converted.

**4.6 Venn Diagrams and the Traditional Standpoint**

Earlier in this chapter we saw how Venn diagrams can be used to represent the content of categorical propositions from the Boolean standpoint. With a slight modification they can also be used to represent the content of categorical propositions from the traditional, or Aristotelian, standpoint. These modified Venn diagrams can then be used to prove the relationships of the traditional square of opposition, and also to test the validity of immediate inferences from the traditional standpoint.

The difference between the Boolean standpoint and the Aristotelian standpoint rests only on universal (A and E) propositions. From the Boolean standpoint, universal propositions have no existential import, but from the Aristotelian standpoint they do have existential import when their subject terms refer to actually existing things. For example, from the Boolean standpoint the statement "All raccoons are pests" does not imply the existence of anything, but from the Aristotelian standpoint it implies the existence of raccoons. Thus, if we are to construct a Venn diagram to represent such a statement from the Aristotelian standpoint, we need to use some symbol that represents this implication of existence.

The symbol that we will use for this purpose is an X surrounded by a circle. Like the X's that we have used up until now, this circled X signifies that something exists in the area in which it is placed. However, the two symbols differ in that the uncircled X represents the positive claim of existence made by particular (I and 0) propositions, whereas the circled X represents the implication of existence made by universal propositions about actually existing things. Such statements may be diagrammed from the Aristotelian standpoint as follows:

A: All S are P E: No S are P

S P S P

In the diagram for the A statement, the left-hand part of the S circle is shaded, so if there are any members of S, they must be in the area where the two circles overlap. Thus, a circled X is placed in the overlap area. In the diagram for the E statement, the overlap area is shaded, so if there are any members of S they must be in the left-hand part of the S circle. Thus, a circled X is placed in this area.

The diagrams for the I and 0 statements are the same from the Aristotelian stand-point as they are from the Boolean:

I: Some S are P O: Some S are P

S P S P

**Proving the Traditional Square of Opposition**

We can now use this modified Venn diagram technique to prove the relations of the traditional square of opposition. Having such a proof is important because up until now these relations have only been illustrated with various examples; they have not been proved. The accompanying figure reproduces the traditional square of opposition together with Venn diagrams that represent the Aristotelian interpretation of the four standard-form propositions.

Let us begin with the contradictory relation. If the A statement is given as true, then the left-hand part of the S circle is empty. This makes the 0 statement false, because it claims that the left-hand part of the S circle is not empty. And if the 0 statement is given as true, then the left-hand part of the S circle is not empty, which makes the A statement false. On the other hand, if the 0 statement is given as false, then

the left-hand part of the S circle is empty. However, given that some members of S exist, they must be in the overlap area. This double outcome makes the A statement true. Also, if the A statement is given as false, then either the left-hand part of the S circle is not empty, or the overlap area is empty (or both). If the left-hand part of the S circle is not empty, then the 0 statement is true. Alternately, if the overlap area is empty, then, given that some members of S exist, they must be in the left-hand part of the S circle, and, once again, the 0 statement is true. Analogous reasoning applies for the relation between the E and I statements.

Next, we turn to the contrary relation. If the A statement is given as true, then the overlap area is not empty, which makes the E statement false. By analogous reasoning, if the E statement is given as true, the overlap area is empty, which makes the A statement false. However, if the A statement is given as false (making the 0 statement true), then the E statement could be either true or false depending on whether or not the overlap area is empty. Thus, in this case the E statement would have logically undetermined truth value. By analogous reasoning, if the E statement is given as false (making the I statement true), the A statement could be either true or false depending on whether or not the left-hand part of the S circle is empty. Thus, the A statement would have logically undetermined truth value.

Turning next to the subcontrary relation, if the I statement is given as false, then the area where the S and P circles overlap is empty. Given that at least one S exists, there must be something in the left-hand part of the S circle, which makes the 0 statement true. By analogous reasoning, if the 0 statement is given as false, there must be something in the overlap area, making the I statement true. But if the I statement is given as true, then the 0 statement could be either true or false depending on whether something exists in the left-hand part of the S circle. Thus, the 0 statement would have undetermined truth value. Similarly, if the 0 statement is given as true, then the I statement could be either true or false depending on whether something exists in the overlap area. Thus, the I statement would have undetermined truth value.

Finally, we consider subalternation. If the A statement is given as true, then some-thing exists in the area where the S and P circles overlap, which makes the I statement true as well. And if the I statement is given as false, then the overlap area is empty, making the A statement false. But if the A statement is given as false (making the 0 statement true), then the I statement could be either true or false depending on whether something exists in the overlap area. Thus, the I statement would have logically undetermined truth value. And if the I statement is given as true, then the A statement could be either true or false depending on whether or not the left-hand part of the S circle is empty. Thus, the A statement would have logically undetermined truth value. Analogous reasoning applies for the subalternation relation between the E and 0 statements.

**Testing Immediate Inferences**

From the Aristotelian standpoint, the modified Venn diagram technique involving circled X's can be used to test immediate inferences. The only requirement is that the subject and predicate terms of the conclusion be the same as those of the premise. Such inferences depend on the square of opposition and do not involve the operations of con-version, obversion, and contraposition. Venn diagrams can also be used to test inferences involving these latter operations, but a further modification must be introduced.

Since any argument that is valid from the Boolean standpoint is also valid from the Aristotelian standpoint, testing the argument from the Boolean standpoint is often simpler. If the argument is valid, then it is valid from both standpoints. But if the argument is invalid from the Boolean standpoint and has a particular conclusion, then it may be useful to test it from the Aristotelian standpoint. Let us begin by testing an inference form for validity:

All A are B.

Therefore, some A are B.

First, we draw Venn diagrams from the Boolean standpoint for the premise and conclusion:

All A are B Some A are B

A B A B

The information of the conclusion diagram is not represented in the premise diagram, so the inference form is not valid from the Boolean standpoint. Thus, noting that the conclusion is particular, we adopt the Aristotelian standpoint and assume for the moment that the subject of the premise (A) denotes at least one existing thing. This thing is represented by placing a circled X in the open area of that circle:

All A are B Some A are B

A B A B

Now the information of the conclusion diagram is represented in the premise diagram. Thus, the inference form is conditionally valid from the Aristotelian standpoint. It is valid on condition that the circled X represents at least one existing thing.

To test a complete inference we begin by testing its form. Here is an example:

No penguins are birds that can fly.

Therefore, it is false that all penguins are birds that can fly.

First, we reduce the immediate inference to its form and test it from the Boolean standpoint:

No A are B It is false that all A are B

A B A B

Since the inference form is not valid from the Boolean standpoint, we adopt the Aristotelian standpoint and assume for the sake of this test that the subject of the premise (P) denotes at least one existing thing:

No A are B It is false that all A are B

A B A B

The Venn diagrams show that the inference form is conditionally valid from the Aristotelian standpoint. It is valid on condition that the circled X represents at least one existing thing. Since the circled X is in the P circle, the final step is to see if the term in the inference corresponding to P denotes something that exists. The term in question is "penguins:' and at least one penguin actually exists. Thus, the condition is fulfilled, and the inference is valid from the Aristotelian standpoint.

Another example:

All sugarplum fairies are delicate creatures.

Therefore, some sugarplum fairies are delicate creatures.

This immediate inference has the same form as the first one we tested. The form is not valid from the Boolean standpoint, but it is conditionally valid from the Aristotelian standpoint:

All S are D Some S are D

S D S D

The final step is to see if the circled X represents at least one existing thing. The circled X is in the S circle and S stands for "sugarplum fairies," which do not exist. Thus, the requisite condition is not fulfilled, and the inference is not valid from the Aristotelian standpoint. The inference commits the existential fallacy.

The steps involved in testing an immediate inference from the Aristotelian stand-point may now be summarized:

1. Reduce the inference to its form and test it from the Boolean standpoint. If the form is valid, proceed no further. The inference is valid from both standpoints.
2. If the inference form is invalid from the Boolean standpoint and has a particular conclusion, then adopt the Aristotelian standpoint and look to see if the left-hand premise circle is partly shaded. If it is, enter a circled X in the unshaded part and retest the form.
3. If the inference form is conditionally valid, determine if the circled X represents something that exists. If it does, the condition is fulfilled, and the inference is valid from the Aristotelian standpoint. If it does not, the inference is invalid, and it commits the existential fallacy from the Aristotelian standpoint.